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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION - 2011

SOLUTIONS FOR CLASS : 12-PCM

Mathematics

1. (C) $xRx \Leftrightarrow 2x^2 - 3x.x + x^2 = 0\ ,\ \forall x \in N$

$\therefore$ R is reflexive.

For x = 1, y = 2, $2x^2 - 3xy + y^2 = 0$

$\therefore$ $\mathbb{R}$ but $2\cdot 2^2 - 3\cdot 2\cdot 1 + 1^2 = 3 \neq 0$

So 2 is not R-related to 1.

$\therefore$ R is not symmetric.

Hence (C) is the correct answer.

2. (B) $\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \frac{1}{1+\tan^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$

$$\therefore \begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \cos^2\theta \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$

$$= \cos^2\theta \begin{bmatrix} 1-\tan^2\theta & -2\tan\theta \\ 2\tan\theta & 1-\tan^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$$

$\therefore$ $a = \cos 2\theta,\ b = \sin 2\theta$

3. (A) $x \neq 0,\ \frac{f(x)}{x^2} = \begin{vmatrix} \cos x & 1 & 1 \\ \frac{2\sin x}{x} & 1 & 2 \\ \frac{\tan x}{x} & 1 & 2 \end{vmatrix}$

(divide each of R_2 and C_2 by x)

$$\therefore \lim_{x \to 0}\frac{f(x)}{x^2} = \begin{vmatrix} 1 & 1 & 1 \\ 2(1) & 1 & 2 \\ 1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = -1$$

(using $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$)

4. (D) $\underset{x\to\infty}{\text{Lt}}\ x^{2n} = \begin{cases} 0 & \text{if } |x| < 1 \\ 1 & \text{if } |x| = 1 \end{cases}$

$$\Rightarrow f(x) = \underset{x\to\infty}{\text{Lim}}\ (\sin x)^{2n} = \begin{cases} 0 \text{ if } |\sin x| < 1 \\ 1 \text{ if } |\sin x| = 1 \end{cases}$$

This shows that f is continuous for all x, except possibly when $|\sin x| = 1$,

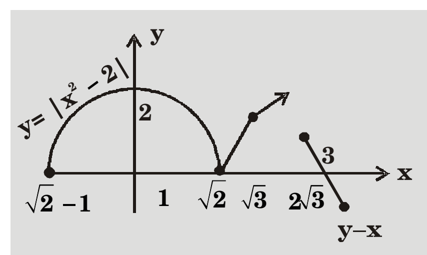
i.e., when $x = (2k+1)\pi/2,\ (k \in I)$

for those points, we have,

$$\underset{x \to (2x+1)\frac{\pi}{2}}{\text{Lim}} f(x) = 0 \neq 1 = f\left((2k+1)\frac{\pi}{2}\right)$$

Hence, f(x) is discontinous.

5. (B) From the graph



Maximum occurs at x = 0 and minimum at x = 4.

6. (A) $\int f(x)dx = f(x),$

$\Rightarrow \quad \frac{d}{dx}(f(x)) = f(x)$

$\Rightarrow \quad \frac{1}{f(x)}d(f(x)) = dx$

on integrating we get,

$\Rightarrow \quad \log(f(x)) = x + \log C$

$\Rightarrow \quad f(x) = Ce^x$

$\Rightarrow \quad \{f(x)\}^2 = C^2e^{2x}$

$$\Rightarrow \quad \int\{f(x)\}^2 dx = \int C^2e^{2x}dx = \frac{C^2e^{2x}}{2}$$

$$= \frac{1}{2}\{f(x)\}^2$$

7. (D) The total number of ways : 7^4.

The number of ways of choosing two teachers out of 7 is ${}^7C_2$.

The number of ways in which they can check four papers is 2^4.

But this includes two ways in which all the papers will be checked by a single teacher. Therefore, the number of ways in which 4 papers can be checked by exactly two teachers is $2^4 - 2 = 14$.

$\therefore$ Number of favourable ways

$$= {}^7C_2 \times 14 = 21 \times 14$$

Required probability $= \dfrac{21 \times 14}{7^4} = \dfrac{6}{49}$

8. (A) Equation of a plane containing the face OAB is $ax + by + cz = 0$

$a + 2b + c = 0$ and $2a + b + 3c = 0$

$\Rightarrow \dfrac{a}{5} = \dfrac{b}{-1} = \dfrac{c}{-3}$ and the direction ratios of the plane are 5, –1, –3.

Similarly plane containing the face ABC are 1, –5, –3.

The required angle,

$$\cos\theta = \frac{5 \times 1 + (-1)(-5) + (-3)(-3)}{\sqrt{5^2 + 1^2 + 3^2}\ \sqrt{1^2 + 5^2 + 3^2}} = \frac{19}{35}$$

9. (A) Area bounded by $y = f(x)$, $y = x$ and the lines $x = 1$, $x = t$ is given by

$$\int_1^t \{f(x) - x\}dx$$

But this area is given equal to

$\left(t + \sqrt{1+t^2} - \sqrt{2} - 1\right)$ Sq. units

$$\therefore \int_1^t \{f(x) - x\}dx = t + \sqrt{1+t^2} - \sqrt{2} - 1, \forall\, t > 1$$

On differentiating both sides w.r.t t, we get

$$f(t) - t = 1 + \frac{t}{\sqrt{1+t^2}}\ \forall\, t > 1$$

$$\Rightarrow f(t) = t + 1 + \frac{t}{\sqrt{1+t^2}}\ \forall\, t > 1$$

Hence $f(x) = x + 1 + \dfrac{x}{\sqrt{1+x^2}}\ \forall\, t > 1$

10. (A) $y^2 = 2cx + 2c^{3/2}$

$2yy' = 2c$ or $c = yy'$

$$\Rightarrow y^2 = 2xyy' + 2yy'\sqrt{yy'}$$

$$\Rightarrow (y - 2xy')^2 = 4y'^2(yy')$$

$$\Rightarrow 4y(y')^3 + 4xyy' - y^2 - 4x^2(y')^2 = 0$$

$\therefore$ Order = 1, degree = 3

11. (C) $f(x) = 0$ if $x \in I$ and for $x \in R \sim I$

$2(x - [x]) < 1 + x - [x]$

Thus $f(x) < \dfrac{1}{2}$.

12. (A) $$\tan^{-1}\left(\frac{1}{1+r+r^2}\right) = \tan^{-1}\left(\frac{r+1-r}{1+r(r+1)}\right)$$

$$= \tan^{-1}(r+1) - \tan^{-1}(r)$$

$$\Rightarrow \sum_{r=0}^{n}\left[\tan^{-1}(r+1) - \tan^{-1}(r)\right]$$

$$= \tan^{-1}(n+1) - \tan^{-1}(0)$$

$$= \tan^{-1}(n+1)$$

$$\Rightarrow \sum_{r=0}^{\infty}\tan^{-1}\left(\frac{1}{1+r+r^2}\right) = \tan^{-1}(\infty) = \frac{\pi}{2}$$

13. (A) Let $B = I + A + A^2 + \ldots.. + A^{k-1}$

$\Rightarrow B(I - A) = (I + A + A^2 + \ldots.. + A^{k-1})(I - A)$

$= I - A^k = I$, Since $A^k = 0$

$\Rightarrow B = (I - A)^{-1}$

Hence, $(I - A)^{-1} = I + A + A^2 + \ldots.. + A^{k-1}$

Thus, $P = -1$

14. (A) $$f''(x) = x^{-1/3} = \frac{d}{dx}\left(\frac{3}{2}x^{2/3} + c\right)$$

So $f'(x) = \dfrac{3}{2}x^{2/3} + \text{const.}$

$$= \frac{d}{dx}\left(\frac{9}{10}x^{5/3} + \text{const.}\right)$$

$$\Rightarrow f(x) = \frac{9}{10}x^{5/3} + \text{const.}$$

15. (C) $f^{11}(x) > 0\ \forall\, x \in R$

$\Rightarrow$ $f^1(x)$ is increasing $x \in R$

$g^1(x) = -f^1(2-x) + f^1(4+x)$

If $g^1(x) > 0 \Rightarrow f^1(4+x) > f^1(2-x)$

$\Rightarrow$ $4 + x > 2 - x$

$\Rightarrow$ $x > -1$.

16. (D) Total number of questions = 25

Student has worked out = 20

The probability that the student can solve both the questions correctly

$$= \frac{{}^{20}C_2}{{}^{25}C_2} = \frac{19}{30}$$

17. (C) Let vertices are A, B, C, D and O is origin

$\therefore \overline{OA} = \hat{i} - 6\hat{j} + 10\hat{k}$

$\overline{OB} = -\hat{i} - 3\hat{j} + 7\hat{k}$

$\overline{OC} = 5\hat{i} - \hat{j} + \lambda\hat{k}$

$\overline{OD} = 7\hat{i} - 4\hat{j} + 7\hat{k}$

$\therefore \overline{AB} = \overline{OB} - \overline{OA} = -2\hat{i} + 3\hat{j} - 3\hat{k}$

$\therefore \overline{AC} = \overline{OC} - \overline{OA} = 4\hat{i} + 5\hat{j} + (\lambda - 10)\hat{k}$

$\overline{AD} = \overline{OD} - \overline{OA} = 6\hat{i} + 2\hat{j} - 3\hat{k}$

Volume of tetrahedron

$= \frac{1}{6}[\overline{AB}\ \overline{AC}\ \overline{AD}]$

$= \frac{1}{6}\begin{vmatrix} -2 & 3 & -3 \\ 4 & 5 & \lambda - 10 \\ 6 & 2 & -3 \end{vmatrix}$

$= \frac{1}{6}\{-2(-15 - 2\lambda + 20) - 3(-12 - 6\lambda + 60) - 3(8 - 30)\}$

$= \frac{1}{6}\{4\lambda - 10 - 144 + 18\lambda + 66\}$

$= \frac{1}{6}(22\lambda - 88) = 11$

$\Rightarrow 2\lambda - 8 = 6$

$\therefore\ \lambda = 7$

18. (D) Putting $v = \frac{y}{x}$ so that $x\frac{dv}{dx} + v = \frac{dy}{dx}$

we have $x\frac{dv}{dx} + v = v + \phi\left(\frac{1}{v}\right)$

$\Rightarrow \quad \frac{dv}{\phi(1/v)} = \frac{dx}{x}$

$\Rightarrow \quad \log|Cx| = \int \frac{dv}{\phi(1/v)}$

But $y = \frac{x}{\log|(Cx)|}$ is the general sol. So

$\frac{x}{y} = \frac{1}{v} = \log|Cx| = \int \frac{dv}{\phi(1/v)}$

$\Rightarrow \ \phi\left(\frac{1}{v}\right) = -v^2$

$\Rightarrow \phi(x/y) = -y^2/x^2$

19. (A) A vector d in the plane of a and b is of the form

$\lambda a + \mu b = (\lambda + \mu)i + (2\lambda - \mu)j + (\lambda + \mu)k$

The projection of d on c is $\frac{c \cdot d}{|c|}$

$= \lambda + \mu + (2\lambda - \mu) - (\lambda - \mu) = \frac{1}{\sqrt{3}}$ if

$2\lambda - \mu = 1$. Thus d is of the form

$(3\lambda - 1)i + j + (3\lambda - 1)k$, so $-4i + j - 4k$ is

d for $\lambda = -1$.

20. (A) $\vec{a}.\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta = 1 \,.\, 1\cos\theta = \cos\theta$

$\therefore \left|\frac{\vec{a} - \vec{b}}{2}\right|^2 = \frac{\left(\vec{a} - \vec{b}\right)^2}{4}$

$= \frac{\left(\vec{a}\right)^2 + \left(\vec{b}\right)^2 - 2\vec{a}.\vec{b}}{4}$

$= \frac{1 + 1 - 2\vec{a}.\vec{b}}{4} = \frac{1 - \cos\theta}{2}$

$= \sin^2\left(\frac{\theta}{2}\right)$

$\left|\frac{\vec{a} - \vec{b}}{2}\right| = \sin\left(\frac{\theta}{2}\right)$

21. (C) $g(f(h(t))) = g(f(4t - 8)) = g\left(\sqrt{4t - 8}\right) = \sqrt{\frac{4t - 8}{4}}$

22. (C) $A = 2\tan^{-1}(2\sqrt{2} - 1)$

$= 2\tan^{-1}(1.828) = A > 2\tan^{-1}\sqrt{3}$

$\Rightarrow A = \frac{2\pi}{3}$

we have $\sin^{-1}\left(\frac{1}{3}\right) < \sin^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow \quad \sin^{-1}\left(\frac{1}{3}\right) < \frac{\pi}{6}$

$\Rightarrow \quad 3\sin^{-1}\left(\frac{1}{3}\right) < \frac{\pi}{2}$

Also $3\sin^{-1}\left(\frac{1}{3}\right) = \sin^{-1}\left[3 \cdot \frac{1}{3} - 4\left(\frac{1}{3}\right)^3\right]$

$= \sin^{-1}\left(\frac{23}{27}\right) = \sin^{-1}(0.882)$

$\Rightarrow \quad 3\sin^{-1}\left(\frac{1}{3}\right) < \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$$\Rightarrow \quad 3\sin^{-1}\left(\frac{1}{3}\right) < \frac{\pi}{3}$$

Also $\sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}(0.6) < \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$$\Rightarrow \quad \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{3}$$

Hence $B = 3\sin^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(\frac{3}{5}\right)$

$$< \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

Hence A > B.

23. (C) f(x) is a polynomial

$f(x) = A_0 + A_1 x + \ldots\ldots A_{232} x^{232}$

The coeff. of x = $f^1(0)$

now differentiating both sides w.r.t x, and x = 0,

$$f^1(0) = \begin{vmatrix} 22 & 44 & 66 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 33 & 66 & 99 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 44 & 88 & 144 \end{vmatrix}$$

$$\Rightarrow f'(0) = 0 + 0 + 0 \Rightarrow f'(0) = 0$$

24. (B) $f(g(x)) =$

$$\frac{1}{\left(\frac{1}{x^2} - 1\right)\left(\frac{1}{x^2} - 2\right)} = \frac{x^4}{(1 - x^2)(1 - 2x^2)}$$

$\Rightarrow f(g(x))$ is discontinuous at

$x = \pm 1, x = \pm\frac{1}{\sqrt{2}}$ and x = 0

Since, g(x) is discontinuous at x = 0.

25. (A) Put $\pi \log x = \theta$, so that

$$I = \int_0^{2007\pi} \cos\theta \, d\theta = [\sin\theta]_0^{2007\pi} = 0$$

26. (A) There are 20 unstamped ordinary envelopes.

∴ There are 30 stamped ordinary envelopes

∴ There are 18 stamped airmail envelopes. So, required probability

$$= \frac{30 - 18}{80} = \frac{12}{80}$$

27. (C) $(x\cos x - \sin x)dx = \frac{x}{y}\sin x \, dy$

$$xy\cos x \, dx - y\sin x dx = x\sin x dy$$

$$xy\cos x \, dx = x\sin x \, dy + y\sin x \, dx$$

$$xy\cos x \, dx = (x\,dy + y\,dx)\sin x$$

$$\cot x \, dx = \frac{x\,dy + y\,dx}{xy} = \frac{d(xy)}{xy}$$

on integrating we get,

$$\log|\sin x| = \log|xy| + \log c$$

(or) $\frac{|\sin x|}{|xy|} = c$

28. (C) b x c = – 3i – j + 2k. If θ is the angle between a and the plane containing b and c

then $\cos(90 - \theta) = \frac{|a.(b \times c)|}{|a||b \times c|}$

$$= \frac{1}{\sqrt{14}}\frac{1}{\sqrt{14}}|(-9 - 2 + 2)| = \frac{9}{14}.$$

Hence, $\theta = \sin^{-1}(9/14)$

29. (D) Taking dot product on both sides by a, b and c, we have

a.a + a.b + a.c = 0, b.a + b.b + b.c = 0,

c.a + c.b + c.c = 0

$\Rightarrow$ 29 + 2 (a.b + b.c + c.a) = 0

$\Rightarrow$ (a.b + b.c + c.a) = – 29/2

30. (B) (2x + 3y + p) (3x + 4y + q)

$= 6x^2 + 17xy + 12y^2 + 22x + 31y + 20$

$\Rightarrow$ 2q + 3p = 22, 3q + 4p = 31, pq = 20

$\Rightarrow$ p + q = 9 and $p^2 + q^2 = 81 - 2 \times 20 = 41$

31. (A) Put $\sqrt{x + 1} = t$ or $x + 1 = t^2$

$$\therefore \quad I = \int_3^4 \frac{2t}{(t^2 - 4)\,t} dt$$

$$= \frac{2}{(2)(2)} \log\left[\left|\frac{t - 2}{t + 2}\right|\right]_3^4$$

$$= \frac{1}{2}\left[\log\frac{1}{3} - \log\frac{1}{5}\right]$$

$$= \frac{1}{2}\log\frac{5}{3}$$

32. (D) $b\sin^{-1}x + b\cos^{-1}x = \frac{b\pi}{2}$

and $a\sin^{-1}x - b\cos^{-1}x = c$

$$\Rightarrow \quad (a+b)\sin^{-1}x = \frac{b\pi}{2} + c$$

$$\Rightarrow \quad \sin^{-1}x = \frac{(b\pi)/2 + c}{a+b}$$

Similarly $\cos^{-1}x = \dfrac{(a\pi)/2 - c}{a+b}$

So that

$$a\sin^{-1}x + b\cos^{-1}x = \frac{\pi ab + c(a-b)}{a+b}$$

33. (D) Let A be a skew symmetric matrix of order n.

Then $A^1 = -A \Rightarrow |A^1| = |-A|$

$$\Rightarrow \quad |A| = (-1)^n |A| \Rightarrow |A| = -|A|$$

[$\because$ n is odd]

$$\Rightarrow \quad 2|A| = 0 \Rightarrow |A| = 0$$

$\therefore$ A^{-1} does not exist.

34. (B) By circulant determinant property

a + b + c = 0

$\Rightarrow$ x + 3 + 6 = x + 2 + 7 = x + 4 + 5 = 0

$\Rightarrow$ x = –9

35. (A) $\dfrac{x^2 - y^2}{x^2 + y^2} = \cos\log a = k$

Put $u = \dfrac{y}{x}$, on componendo & dividendo

$$\left(\frac{y}{x}\right)^2 = u^2 - \frac{(1-A)}{1+A}$$

$$\Rightarrow \quad \frac{y}{x} = \sqrt{\frac{1-A}{1+A}} \Rightarrow x\frac{dy}{dx} - y = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}.$$

36. (D) Differentiating w.r.t.x,

$$3y^2\frac{dy}{dx} + 6x = 12\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-2x}{y^2 - 4}$$

At pt. where the tangent(s) is/are vertical, $\dfrac{dy}{dx}$ is not defined i.e., at these pts,

$y^2 - 4 = 0 \Rightarrow y = \pm 2$

when y = 2, $8 + 3x^2 = 12(2)$

$\Rightarrow 3x^2 = 16 \Rightarrow x = \pm\dfrac{4}{\sqrt{3}}$

when y = –2, $-8 + 3x^2 = -24$

$\Rightarrow 3x^2 = -16$

This is not possible.

Thus, the required pts. are $\left(\pm\dfrac{4}{\sqrt{3}}, 2\right)$.

37. (A) Put $t = \dfrac{1}{2}(x - 1 + x + 2) = x + \dfrac{1}{2}$

So that dx = dt, $5 + 3x = \dfrac{7}{2} + 3t$ and

$$(x-1)(x+2) = \left(t - \frac{1}{2} - 1\right)\left(t - \frac{1}{2} + 2\right)$$

$$= t^2 - \left(\frac{3}{2}\right)^2$$

$$\therefore \quad I = \int \frac{3t + \frac{7}{2}}{\sqrt{t^2 - \left(\frac{3}{2}\right)^2}}\,dt = 3\sqrt{t^2 - \left(\tfrac{3}{2}\right)^2}$$

$$+\frac{7}{2}\log\left|t + \sqrt{t^2 - \left(\tfrac{3}{2}\right)^2}\right| + c$$

$$= \sqrt[3]{(x-1)(x+2)} + \frac{7}{2}\log\left|x + \frac{1}{2} + \sqrt{(x-1)(x+2)}\right| + c$$

38. B) Required area

$$= \left|\int_0^1 (x-1)(x-2)(x-3)dx\right| + \left|\int_1^2 (x-1)(x-2)(x-3)dx\right|$$

$$+ \left|\int_2^3 (x-1)(x-2)(x-3)dx\right|$$

$$= \frac{9}{4} + \frac{1}{4} + \frac{1}{4} = \frac{11}{4} \text{ sq unit.}$$

39. (C) Let P (a, b, c) be the foot of the perpendicular from the origin to the plane, then direction ratios of OP are a – 0, b – 0, c – 0 i.e., a, b, c.

So the equation of the plane passing through P (a, b, c) the direction ratios of the normal to which are a, b, c is a (x – a) + b(y – b) + c (z – c) = 0

$\Rightarrow$ ax + by + cz = $a^2 + b^2 + c^2$.

40. (B) $\left|a\hat{i} + b\hat{j} + c\hat{k}\right| = |a| + |b| + |c|$

$$\Rightarrow \quad \sqrt{a^2 + b^2 + c^2} = |a| + |b| + |c|$$

$$\Rightarrow \quad a^2 + b^2 + c^2 = a^2 + b^2 + c^2$$
$$+ 2(|a||b| + |b||c| + |c||a|)$$

$$\Rightarrow \quad |a||b| + |b||c| + |c||a| = 0$$

$\Rightarrow$ ab = bc = ca = 0

Any two of a, b, c are zero.

Physics

41. (C) The total electric flux coming out of the cube is zero. As the diopoles are placed inside the cube and the electric field inside, a body is zero (According to Gaus's law). The total electric flux is zero when electric field is zero.

42. (D) The strength of an electro magnet depends on the voltage applied and the number of turns of the coil. In option (D), the electromagnet has high voltage applied with more number of turns than other electromagnets in option A, B and C.

43. (C) The earth's atmosphere is transparent to visible light and radio waves, but absorbs X-rays. Therefore, X-ray astronomy is possible only from satellites orbiting the earth.

44. (B) In a step-up transformer, $V_o > V_i$, same period and V_0 not is 180^0 out of phase with V_i. So option (B) satisfies these conditions.

45. (C) Critical angle for the liquid-air surface is

$$1.6 \sin C = 1$$

$$\Rightarrow \sin C = \frac{1}{1.6} = 0.625$$

$$\sin i = 0.7$$

$\therefore i > C$, hence ray will be internally reflected.

46. (B) Let I_1 be the current through $6\,\Omega$ resistance I_2 through $(9 + 3)\ \Omega$ resistance.

$$I_1^2 \times 6 = 24$$

$$I_1 = 2\text{ A}$$

P.D. across C and D = 2×6 = 12 V

$$\text{Current } I_2 = \frac{12}{9+3} = 1\text{A}$$

$\therefore$ Total current I through $2\,\Omega$

$$= (2 + 1) = 3\text{ A}$$

Heat produced per second in $2\,\Omega$

$$= I^2R = 3^2 \times 2 = 18 \text{ cal s}^{-1}$$

47. (B) Mass of ${}_6C^{12}$ mass of neutron

$$= \text{total mass of } {}_6C^{13}$$

$$= 12 + 1.00867 = 13.00867 \text{ U}$$

Atomic mass of ${}_6C^{13}$ = 13.00335 U

Mass defect = 13.00867 – 13.00335

$$= 0.0532 \text{ U}$$

B.E. of least bound neutron in a ${}_6C^{13}$ nucleus (in MeV) = 0.0532×931

$$= 4.953 \text{ MeV}$$

48. (D) NAND gate is also called as universal gate as it can be used to construct any kind of gate. Hence, forms a base for the digital circuits.

49. (B) In Fresnel diffraction, the ray from source is not parallel to the screen, while both source and screen are near the aperture. But in Fraunhoffer diffraction both light source and screen are very far from the aperture and ray incident on the aperture and ray leaving the aperture are parallel.

50. (A) Pulse code modulation uses digital signals for modulating the information signals.

51. (B) Here, I series $\dfrac{2E}{4+2r} = \dfrac{E}{4+\frac{r}{2}}$

$$8 + r = 4 + 2r$$

$$r = 4\,\Omega$$

52. (A) The shortest wavelength of X-rays photon is given by:

$$\lambda_0 = \frac{hc}{eV}$$

$$= \frac{(6.6\times10^{-34}\text{J s})(3.0\times10^{8}\text{ m/sec})}{(1.6\times10^{-19}\text{C})(5.0\times10^{4}\text{V})}$$

$$= 2.5\times10^{-11}\text{ m}$$

53. (C) D.C. is blocked by a capacitor. So, bulb does not light up when energy source is dc for circuit in option (C).

54. (A) Here, $8\times\frac{4}{3}\pi(r)^3 = \frac{4}{3}\pi R^3$

$$r = 1 \text{ mm}$$

or $\quad R = 2r = 2$ mm

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} = \frac{9\times10^9\times8\times0.066\times10^{-12}}{2\times10^{-3}}$$

$$= 2.4 \text{ volts}$$

55. (B) Given M = 30 i.e. $\dfrac{I_c}{I_b} = 30$

$I_e = I_b + I_c$

Divide the equation by I_b

$$\frac{I_e}{I_b} = 1 + \frac{I_c}{I_b} = 1 + 30 = \frac{31}{1} \text{ (or) } 31 : 1$$

56. (C) $I \ \alpha \ A^2$ (A = amplitude)

$$\frac{I_1}{I_2} = \frac{9}{1}$$

$$\Rightarrow \frac{A_1}{A_2} = \sqrt{\frac{I_1}{I_2}} = 3$$

$$\Rightarrow \frac{A_1 + A_2}{A_1 - A_2} = \frac{4}{2} = \frac{2}{1}$$

$$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = 4:1$$

57. (A) Under minimum deviation condition,

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

If $\delta_m = A$, then

$$\Rightarrow \mu = \frac{\sin\left(\frac{A+A}{2}\right)}{\sin\frac{A}{2}} = \frac{\sin A}{\sin\frac{A}{2}} = 2\cos\frac{A}{2}$$

$$A = 2\cos^{-1}\left(\frac{\mu}{2}\right)$$

58. (B) At night in the absence of suns radiation no ionisation can take place. Moreover, the recombination of ions takes place. Hence, the E-layer of the ionosphere disappears at night.

59. (C) Field produced by X and Z at position of Y is:

$$B_Y = B_X - B_Z = \frac{\mu_0 \times 3}{2\pi \times 3 \times 10^{-2}} - \frac{\mu_0 \times 2}{2\pi \times 4 \times 10^{-2}}$$

$$= 10^{-5}\,T$$

$F = B_Y\, il \sin 90^0$

$= 10^{-5} \times 1 \times 0.5$

$= 5 \times 10^{-6}\,N$

Since B_Y is perpendicular to plane of paper and downwards, hence, it acts left to right.

60. (A) According to Maxwell's hypothesis, an accelerated charge produces a sinusoidal time varying magnetic field, which in turn produces a sinusodial time varying electric field. The two fields so produced are mutually perpendicular and are sources at each other.

61. (B) We have $u = f + x$ and $v = f + y$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{1}{f+x} + \frac{1}{f+y}$$

$$= \frac{f+y+f+x}{(f+x)(f+y)}$$

(or) $(f + x)(f + y) = f(2f + x + y)$

(or) $f^2 + fx + fy + xy = 2f^2 + fx + fy$

(or) $xy = f^2$

62. (D) $R_1 = 5R$

$R_2 = R + R + R = 3R$

A balanced wheatstone bridge is in series with two resistance R and R.

$$\therefore \frac{R_2}{R_1} = \frac{3R}{5R} = 3:5$$

63. (B) Since the two capacitors by K_1 and K_2 are in parallel, therefore

$$C_P = C_1 + C_2$$

$$= \frac{\epsilon_0 AK_1}{2t} + \frac{\epsilon_0 AK_2}{2t}$$

$$= \frac{\epsilon_0 A}{2t}(K_1 + K_2)$$

64. (A) $Bil = mg$

$B \times 9.8 \times 1 = 13 \times 10^{-3} \times 9.8$

$B = 3 \times 10^{-3}\,T$

65. (C) A zener can operate either in forward or reverse biased condition. When zener breakdown occurs the applied voltage is greater than zener breakdown voltage. The current in the zener region is in opposite direction to that of the forward biased diode. Zener diode cannot be used as a half wave rectifier.

Chemistry

66. (C) Contribution of A atoms in the corner

$$= \frac{1}{8} x 7 = \frac{7}{8}$$

Contribution of B atoms the face centre

$$= 6 x \frac{1}{2} = 3$$

The formulae $= A_{7/8}B_3 = A_7B_{24}$

67. (C) i) Due to missing of an atom in the unit cell leads to Frenkel defect.

ii) In metal excess defects, A negative ion may be missing from its lattice site leaving a 'hole' which is occupied by an extra electron to maintain the electrical neurality. The anionic site, occupied by an electron is called F-centres.

68. (A) $$\Delta T = \frac{1000 \times k_f \times w}{M.wt \times W}$$

$$0.45 = \frac{1000 \times 5.12 \times 0.2}{20 \times M.wt} \Rightarrow M.wt = 113.8$$

$$2CH_3COOH \rightleftharpoons (CH_3COO)_2$$

Before association $\quad 1 \quad 0$

After association $\quad 1-\alpha \quad \frac{\alpha}{2}$

$$\frac{m_{(Normal)}}{m_{(Observed)}} = 1 - \alpha + \frac{\alpha}{2}$$

$$= \frac{60}{113.8} = 1 - \alpha + \frac{\alpha}{2} \Rightarrow \alpha = 0.945$$

69. (B) Reducing agent means tendency to oxidise; Among the elements Zn possess less reduction potential value. Hence it is a strong reducing agent.

70. (B) $K = C \times \frac{l}{a}$

$\frac{l}{a} = \frac{k}{c} = 0.0212 \times R$

$= 0.0212 \times 55$

$= 1.166\ cm^{-1}$

71. (D) In 1 hour decrease in conc. of $A = \frac{0.6 - 0.5}{1} = 0.1$

In 1 hour increase in conc. of $B = \frac{0.2 - 0}{1} = 0.2$

(i) 0.1 mole A gives 0.2 mole B in a given time thus, n = 2

(ii) $K_C = \frac{[B]^n}{[A]} = \frac{(0.6)^2}{0.3} = 1.2$ mole/lit.

72. (C) For a first order reaction :

$-2.303 \log(a - x) = Kt - 2.303 \log a$

$\log(a - x) = \frac{-K}{2.303} t + \log a$ slope $= \frac{-K}{2.303}$

73. (A) Physical adsorption is inversely proportional to temperature because physical adsorption decreases as the temperature increases. Hence, the graph between $\frac{x}{m}$ versus temperature is a rectangular hyperbola.

74. (B) The formation of Carbon monoxide from carbon can reduce the metal oxide to metal at a temperature of the point of intersection of the two lines. In the given figure the temperature 200.

75. (C)

$$Cl_2 + H_2O \rightarrow HCl + HClO$$
$$\downarrow$$
$$H^+Cl^-$$

$$AgNO_3 + Cl^- \rightarrow AgCl + NO_3^-$$

$$2HCl + Mg \rightarrow MgCl_2 + H_{2(g)}$$

76. (C) The compound is Asprin, which is used as analgesic.

77. (A) Higher the Electronegativity of the halogen atom, greater is the acid strength of the acid.

78. (C) A, B, D are the correct uses of the helium but helium is not used in the preparation of super conducting magnets.

79. (D) In $S_2O_7^{2-}$; $S-O-S$ bond is present.

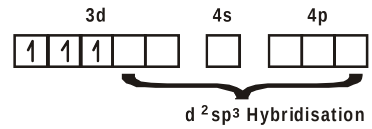
80. (D) $2MnO_4^- + 6H^+ + 5(COOH)_2 \rightarrow 2Mn^{+2} + 10CO_2 + 8H_2O$

The Mn^{+2} ions that are formed in the above reaction acts as catalyst and enhances the rate of reaction.

81. (D) $[Pt(NH_3)_6]Cl_4 \rightleftharpoons [Pt(NH_3)_6]^{4+} + 4Cl^-$

Hence, $[Pt(NH_3)_6]Cl_4$ is a complex compound that furnishes ions in solution while $[PtCl_2(NH_3)_2]$ does not furnish ions in solution. So, $[Pt(NH_3)_6]Cl_4$ is an electrical conducting medium in aqueous solution.

82. (C) EC of Cr^{+3} in ground state:



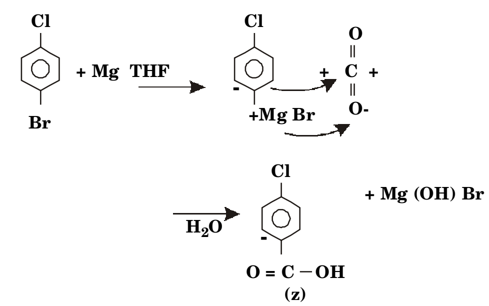
$\mu \sqrt{n(n+2)}$

$= \sqrt{3(3+2)} = \sqrt{(15)} = 3.87$ BM

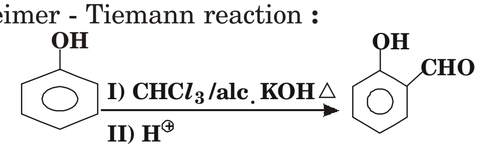
83. (A) $H_2NH_2 + CHCl_3 + 3KOH \rightarrow C_2H_5NC(A) +$

$3KCl(B) + 3H_2O$ Ethyl isocyanide

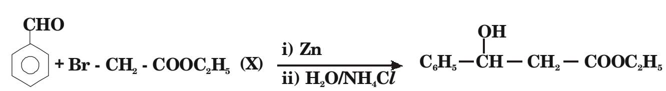
84. (A)



85. (D) Reimer - Tiemann reaction :



86. (C)



87. (D)

$CH_3COOH \xrightarrow{LiAlH_4} C_2H_5OH$

$CH_3COOH \xrightarrow{HI + Red\ P} CH_3 - CH_3$

A – $LiAlH_4$, B – HI + Red P

88. (C)

$$R - NH_2 \xrightarrow{HNO_2} R - \overset{\oplus}{N} \equiv N \rightarrow R^+ + N_2$$
$$\downarrow H_2O$$
$$R - OH$$

89. (B) The linkage between two glucose moities in amylose is α, and the linkage between two glucose moities in cellulose is β.

90. (D)

H
H – C – COOH The central carbon is not a symmetric.
NH₂

Glycine is an optically inactive amino acid.