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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION - 2011

SOLUTIONS FOR CLASS : 11-PCM

Mathematics

1. (D) P = {1, 2} or {1, 2, 3} or {1, 2, 4} or {1, 2, 5} or {1,2,3,4} or (1,2,3,5} or {1,2,4,5} or {1,2,3,4,5}

2. (D) Since A is not the union of A_1, A_2 and A_3 the sets A_1, A_2, A_3 do not form a partition of A.

3. (C) The given relation can be written as

$$\sin x\left(1+\sin^2 x\right)=1-\sin^2 x=\cos^2 x$$

$$\sin x\left(2-\cos^2 x\right)=\cos^2 x$$

squaring on both sides

$$\sin^2 x\left(2-\cos^2 x\right)^2=\cos^4 x$$

$$\left(1-\cos^2 x\right)\left(4-4\cos^2 x+\cos^4 x\right)=\cos^4 x$$

$$\cos^6 x-4\cos^4 x+8\cos^2 x=4$$

4. (D) Range = greatest no - smallest no

21 = 25 – b

b = 25 – 21 = 4

5. (C) We have, for $Z \in C$

$|2i| = |Z + (2i - Z)| \le |Z| + |2i - Z|$

$2 \le |Z| + |Z - 2i|$

Thus maximum value of $|Z| + |Z - 2i|$ is 2 and it is attained for any Z lying on the segment joining Z = 0 and Z = 2i.

6. (B) $\operatorname{cosec}\theta+\cot\theta=-\dfrac{b}{a}$, $\operatorname{cosec}\theta\cot\theta=\dfrac{c}{a}$

As $\operatorname{cosec}^2\theta-\cot^2\theta=1$, we get

$\operatorname{cosec}\theta-\cot\theta=-\dfrac{a}{b}$.

$$\therefore \operatorname{cosec}\theta=-\frac{1}{2}\left(\frac{b}{a}+\frac{a}{b}\right)$$

and $\cot\theta=\dfrac{1}{2}\left(\dfrac{a}{b}-\dfrac{b}{a}\right)$

Thus, $-\dfrac{1}{4}\left(\dfrac{a^2}{b^2}-\dfrac{b^2}{a^2}\right)=\dfrac{c}{a}$

$$\Rightarrow a^4-b^4=-4acb^2$$

$$\Rightarrow a^4=b^2\left(b^2-4ac\right)=b^2\Delta$$

7. (A) Using AM > GM, we have

$$\frac{(s-b)+(s-c)+(s-d)}{3}>\left\{(s-b)(s-c)(s-d)\right\}^{1/3}$$

$$\Rightarrow 3s-(b+c+d)>3\left\{(s-b)(s-c)(s-d)\right\}^{1/3}$$

$a>3\left\{(s-b)(s-c)(s-d)\right\}^{1/3}$, similarly we have

$b>3\left\{(s-c)(s-d)(s-a)\right\}^{1/3}$ and

$c>3\left\{(s-a)(s-b)(s-d)\right\}^{1/3}$ and

$d>3\left\{(s-a)(s-b)(s-c)\right\}^{1/3}$, multiplying we get

$$abcd>81(s-a)(s-b)(s-c)(s-d)$$

8. (D) We have,

$$E=\frac{31!}{2^{31}(32!)}=\frac{1}{2^{31}(32)}=\frac{1}{2^{36}}$$

$$=2^{-36}=\left(2^3\right)^{-12}=8^{-12}$$

Thus, x = – 12

9. (C) Note that $5^{5^{5^{\cdot^{\cdot^{5}}}}}$ (23 times 5) is an odd number which is natural.

$$\therefore x=5^{2m+1}=\left(25^m\right)5$$

Where m is a natural number. Thus $x = (24 + 1)^m\, 5 = 5 +$ a multiple of 24. Hence the desired number is 5.

10. (A) ${}^5C_1+{}^5C_2+{}^5C_3=25$

11. (C) Let O be the centre of the circle of unit radius and the coordinates of A_0 be (1, 0) coordinates of A_1 are

$$\left(\cos 60^0, \sin 60^0\right)=\left(1/2, \sqrt{3}/2\right)$$

A_3 are (–1, 0)

A_4 are $\left(-1/2, -\sqrt{3}/2\right)$ and A_5 are $\left(1/2, -\sqrt{3}/2\right)$

$A_0A_1 = 1$

$A_0A_2 = \sqrt{3} = A_0A_4$

so that $(A_0A_1)(A_0A_2)(A_0A_4) = 3$

12. (D) $A_1B_1 = \sqrt{4+4} = 2\sqrt{2}$

$AB = 2\sqrt{2} - 2 = 2\left(\sqrt{2}-1\right)$

Thus equation of required circle is

$x^2 + y^2 = \left(\sqrt{2}-1\right)^2 = 3 - 2\sqrt{2}$

13. (B) $\tan 3\alpha \cot\alpha = \dfrac{3\tan\alpha - \tan^3\alpha}{\tan\alpha\left(1-3\tan^2\alpha\right)}$

$$= \frac{3-\tan^2\alpha}{1-3\tan^2\alpha} = x$$

$$\tan^2\alpha = \frac{x-3}{3x-1} = \frac{(3x-1)(x-3)}{(3x-1)^2}$$

since $\tan^2\alpha$ is non negative, either $x < 1/3$ or $x \geq 3$ so x cannot lie between 1/3 and 3.

14. (A) Let A(5,2,4), B(6,–1,2), C(8,–7,k) be the given points

Direction ratios of AB are

6 – 5, –1 –2, 2 – 4 (i.e.) 1, –3, –2

Direction ratios of BC are

8–6, –7 + 1, k –2 (i.e.) 2, –6, k–2

Since A, B, C are collinear

$$\frac{2}{1} = \frac{-6}{-3} = \frac{k-2}{-2}$$

$k - 2 = -4 \Rightarrow k = 2 - 4 = -2$

15. (D) Since the numerator tends to 0 as $x \to 0$ so

$$\lim_{x\to 0}\frac{1}{x^2}\left(e^{\alpha x} - e^x - x\right) = \frac{1}{2}\lim_{x\to 0}\left(\frac{\alpha e^{\alpha x} - e^x - 1}{x}\right)$$

for the last limit to exist we must have

$$\lim_{x\to 0}\left(\alpha e^{\alpha x} - e^x - 1\right) = 0$$

$\therefore \alpha - 1 - 1 = 0 \Rightarrow \alpha = 2$. For $\alpha = 2$, the last limit is equal to $\dfrac{1}{2}\lim_{x\to 0}\left(\dfrac{2e^{2x} - e^x - 1}{x}\right)$

$$= \frac{1}{2}\lim_{x\to 0}\left(4e^{2x} - e^x\right) = \frac{3}{2}$$

16. (B) $\dfrac{-2(\sin a + \sin b)}{2\sin(a-b)} \times \dfrac{2(\cos a + \cos b)}{2\cos(a-b)} = -1$

$2(\sin a + \sin b)(\cos a + \cos b)$

$= 2\sin(a-b)\cos(a-b)$

$\sin 2a + \sin 2b = \sin 2(a-b) - 2\sin(a+b)$

17. (A) Let the edges be

$\dfrac{a}{r}$, a, ar where r > 1

$\dfrac{a}{r}.a.ar = 216$, i.e., $a^3 = 216$, i.e., a = 6 and

$$2\left(\frac{a}{r}.a + a.ar + \frac{a}{r}.ar\right) = 252;$$

$$\therefore \frac{1}{r} + r + 1 = \frac{7}{2} \Rightarrow r = \frac{1}{2}, 2$$

$\therefore$ a = 6, r = 2, so the longest side = ar = 12

18. (B) Total number of coins = 2 + 2 + 3 + 1 = 8

2 coins are of 10 paise, 2 are of 20 paise, 3 are of 25 paise and 1 is of 50 paise

$\therefore$ required number of ways

$$= \frac{8!}{2!\times 2!\times 3!\times 1!} = 8\times 7\times 6\times 5 = 1680$$

19. (C) Let the equation of the ellipse be $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. As it passes through (4, –1)

$\dfrac{16}{a^2} + \dfrac{1}{b^2} = 1$ (or) $a^2 + 16b^2 = a^2b^2$

since x +4y – 10 = 0 touches the ellipse

$$\left(\frac{10}{4}\right)^2 = a^2\left(\frac{1}{4}\right)^2 + b^2 \Rightarrow a^2 + 16b^2 = 100$$

$a^2b^2 = 100$ and the required equation can be $\dfrac{x^2}{20} + \dfrac{y^2}{5} = 1$

20. (D) $t_7 = {}^nC_6\left(2^{1/3}\right)^{n-6}\left(3^{-1/3}\right)^6$

and $t_{n-5} = {}^nC_{n-6}\left(2^{1/3}\right)^6\left(3^{-1/3}\right)^{n-6}$

Given $t_7 : t_{n-5} = 1 : 6$

$$\Rightarrow \frac{\left(2^{1/3}\right)^{n-6}\left(3^{-1/3}\right)^6}{\left(2^{1/3}\right)^6\left(3^{-1/3}\right)^{n-6}} = \frac{1}{6}$$

$$\Rightarrow \left(2^{1/3}.3^{1/3}\right)^{n-12} = 1/6 = 6^{-1}$$

= n = 9

21. (B) Replacing x by $\dfrac{1}{x}$ in the given equation we get,

$$2f\left(\frac{1}{x^2}\right) + 3f\left(x^2\right) = \frac{1}{x^2} - 1 = \frac{1-x^2}{x^2}$$

solving the given functional equation and the last equation

we have

$$9f\left(x^2\right) - 4f\left(x^2\right) = \frac{3\left(1-x^2\right)}{x^2} - 2\left(x^2-1\right)$$

$$= \left(1-x^2\right)\left(\left(3/x^2\right)+2\right) = \frac{\left(1-x^2\right)\left(3+2x^2\right)}{x^2}$$

$$f\left(x^2\right) = \frac{\left(1-x^2\right)\left(3+2x^2\right)}{5x^2}$$

22. (D)

23. (B) Since, (2 cos x – 1) (3 + 2 cos x) = 0

since $\cos x \neq -\frac{3}{2}$

therefore, $\cos x = \frac{1}{2}$

$$x = 2n\pi \pm \frac{\pi}{3}, n \in I$$

and given $0 \leq x \leq 2\pi$

$\therefore x = \frac{\pi}{3}, \frac{5\pi}{3}$ (for n = 0, 1)

24. (D) $x-\frac{7}{2}, x-3, x-\frac{5}{2}, x-2, x-\frac{1}{2}, x+\frac{1}{2},$

$x+4, x+5$

Median $= \frac{1}{2}\left\{(x-2)+\left(x-\frac{1}{2}\right)\right\} = x-\frac{5}{4}$

25. (A)

26. (B) $P(A \cup B) = 0.6$

$P(A \cap B) = 0.2$

$P(\overline{A}) + P(\overline{B}) = [1-P(A)] + [1-P[B]]$

$= 2-[P(A)+P(B)]$

$= 2-[P(A\cup B)+P(A\cap B)]$

= 2 – 0.6 – 0.2

= 1.2

27. (C) Let first term = a, common ratio = r, where –1 < r < 1

Then, $\frac{a}{1-r} = 2$ and $\frac{a^3}{1-r^3} = 24$

$\therefore \frac{1-r^3}{(1-r)^3} = \frac{1}{3}$

i.e., $1 - 2r + r^2 = 3(1 + r + r^2)$

or $2r^2 + 5r + 2 = 0$

$\therefore r = -2$ or $-\frac{1}{2}$

As –1 < r < 1

$\therefore$ we have $r = -\frac{1}{2}$

Putting this value of r, we get a = 3

$\therefore$ The series is $3-\frac{3}{2}+\frac{3}{4}-\frac{3}{8}\ldots$

28. (A) Here, $8! = 2^7 . 3^2 . 5^1 . 7^1$

obviously, the factors are not multiple of either 2 or 3. So the factors may be 1, 5, 7, 35 of which 5 and 35 are of the form 3m + 2.

So, the sum = 40

29. (C) We have,

$a_r = a_{2n-r}$ for $0 \leq r \leq 2n$

$$\Rightarrow \sum_{r=0}^{n-1} a_r = \sum_{r=0}^{n-1} a_{2n-r}$$

$$\Rightarrow a_0 + a_1 + ...a_{n-1} = a_{2n} + a_{2n-1} + ...a_{n+1}$$

$$\Rightarrow 2(a_0 + a_1 + ...a_{n-1}) + a_n = 3^n \quad \left[\because \sum_{r=0}^{2n} a_r = 3^n\right]$$

$$\Rightarrow a_0 + a_1 + ... + a_{n-1} = \frac{1}{2}\left(3^n - a_n\right)$$

30. (C) $\lim_{x\to 2} \frac{xf(x)-2f(x)}{x-2}$

$= \lim_{x\to 2} \frac{xf(2)-2f'(x)}{1}$

(using L-hospitals rule)

= f(2) – 2 f' (2)

$= 4 - 2 \times 4$

= – 4

31. (D) We have $\alpha = 120^0$ and $\beta = 60^0$

$\therefore \cos\alpha = \cos 120^0 = -\frac{1}{2}$

and $\cos\beta = \cos 60^0 = \frac{1}{2}$

But $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\gamma = 1$$

$$\cos^2\gamma = 1-\frac{1}{4}-\frac{1}{4} = \frac{1}{2}$$

$$\cos\gamma = \pm\frac{1}{\sqrt{2}}$$

$\therefore \gamma = 45^0$ or 135^0

32. (A) Let the middle point of the chord is (h, k)

$\therefore \quad T = S_1$

3xh – 2yk + 2(x + h) — 2k² + 4h – 6k

slope of this chord $= \frac{3h+2}{2k+3} = 2$(given)

or 3h + 2 = 4k + 6

$\Rightarrow$ 3h – 4k = 4

Hence, locus of middle point is

3x – 4y = 4

33. (B) Clearly $a \neq 0$

$$\cot\alpha + \tan\alpha = m \Rightarrow 1 + \tan^2\alpha = m\tan\alpha$$

$\Rightarrow \sec^2 2 = m\tan\alpha$.......(1)

and $\frac{1}{\cos\alpha} - \cos\alpha = n \Rightarrow \sec^2\alpha - 1 = n\sec\alpha$

$\Rightarrow \tan^2\alpha = n\sec\alpha \Rightarrow \tan^4\alpha = n^2\sec^2\alpha$

$\Rightarrow \tan^4\alpha = n^2 m\tan\alpha$ [by (1)]

$\Rightarrow \tan^3\alpha = n^2 m \Rightarrow \tan\alpha = \left(n^2 m\right)^{1/3}$

$\sec^2\alpha = m\left(n^2 m\right)^{1/3}$ [by (1)]

Now $\sec^2\alpha - \tan^2\alpha = 1$

$\Rightarrow m\left(n^2 m\right)^{1/3} - \left(n^2 m\right)^{2/3} = 1$

$\Rightarrow m\left(mn^2\right)^{1/3} - n\left(nm^2\right)^{1/3} = 1$

34. (B) $S_n = (2-1) + \left(2-\frac{1}{2}\right) + ... + \left(2-\frac{1}{n}\right)$

$= 2n - H_n$

35. (B) Normal at $(at^2, 2at)$ on the parabola $y^2 = 4ax$ is $y + tx = 2at + at^3$

Suppose normal equation (i) cuts the curve again at $\left(at_1^2, 2at_1\right)$, then

$2at_1 + att_1^2 = 2at + at^3$

$\Rightarrow 2a\left(t - t_1\right) + at\left(t^2 - t_1^2\right) = 0$

or $2 + t(t + t_1) = 0$

$(\therefore a(t - t_1) \neq 0)$

$\therefore \quad t_1 = -t - \frac{2}{t}$

$= -\left(t + \frac{2}{t}\right)$

36. (B) Required line is $ax + by + K = 0$

$\therefore$ it passes through (c,d)

$\Rightarrow ac + bd + K = 0$

$\Rightarrow K = -ac - bd$

$\therefore$ The equation of the line is

$ax + by - ac - bd = 0$

$a(x - c) + b(y - d) = 0$

37. (B) $2S = a + b + c \Rightarrow S = \frac{1}{2}(a + b + c)$

$S - a = \frac{1}{2}(b + c - a), S - b = \frac{1}{2}(a + c - b)$

and $S - c = \frac{1}{2}(a + b - c)$

$A^2 = S(S-a)(S-b)(S-c)$

$\leq \frac{1}{16}(a + b + c)\frac{1}{27}(a + b + c)^3$

$\therefore A \leq \frac{(a+b+c)^2}{4\times 3\sqrt{3}} = \frac{(2S)^2}{4\times 3\sqrt{3}} = \frac{S^2}{3\sqrt{3}}$

$\therefore A \leq \frac{S^2}{3\sqrt{3}}$

38. (D) As, $Z_1 = \frac{\lambda Z_2 + Z_3}{\lambda + 1}$

Which show Z_1, divides Z_2, Z_3 in the ratio of $1 : \lambda$

Thus the points are collinear

$\therefore$ Distance of A from line BC is zero.

39. (B) We have, $(A \cup B) \cap B' = A - (A \cap B')$

since A and B are disjoint

$\therefore A \cap B = \phi$

$\Rightarrow (A \cup B) \cap B' = A$

$\therefore \left((A \cup B) \cap B'\right) \cup A' = A \cup A' = N$

40. (C) If $x \in Q$, then $f(x) = x$

$\therefore$ $fof(x) = f\{f(x)\} = f(x) = x$

If $x \notin Q$, then $f(x) = 1 - x$

$fof(x) = f\{f(x)\} = f(1 - x) = 1 - (1 - x) = x$

$(\therefore x \notin Q \Rightarrow 1 - x \notin Q)$

Thus, $fof(x) = x$ for all $x \in [0,1]$

Physics

41. (B) Distance travelled in last 3 seconds

$=$ area of ΔBEC

$= \frac{1}{2}\times EC \times BE$

$= \frac{1}{2}\times 3 \times 24$

$= 36$ m

Distance travelled in 8 s = area of ΔOAD + area rectangle $\Delta ABED$ + area of ΔBEC

$= \left(\frac{1}{2}\times 3\times 24\right) + (24\times 2) + (36)$

$= 36 + 48 + 36 = 120$ m

Ratio = 36 : 120 = 3 : 10

42. (B) As area $=$ length $\times$ breadth

$= 4.321 \times 4.055$

Area $= 17.521655$

The area of the sheet upto four significant figures is $17.52\ m^2$

43. (B) Mass of a particle = m

Force = p

Time = t

Here, initial velocity u = 0

p = ma where a is the acceleration

or $a = \frac{p}{m}$

using, $s = ut + \frac{1}{2}at^2$

$s = 0 \times t + \frac{1}{2}\left(\frac{p}{m}\right)t^2$

$s = \frac{1}{2}\frac{pt^2}{m}$

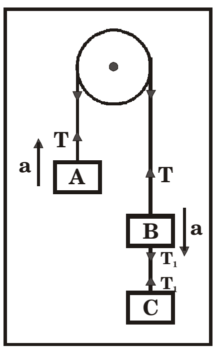
According to work energy theorem,

Kinetic energy = $p \times S$

$= p\left(\frac{1}{2}\frac{pt^2}{m}\right)$

$= \frac{p^2t^2}{2m}$

44. (B) Let T the tension in the string. Let the bodies B and C accelerate downwards with acceleration a. Then the body A move upwards with acceleration a.



Therefore, For motion of body A,

$T - mg = ma$ .... (i)

For motion of body B and C,

$1\ mg - T = 2\ m \times a$ .... (ii)

Adding (i) and (ii), we get

$2\ mg - mg = 3\ ma$ or $a = g/3$

For motion of body C, $mg - T_1 = ma$

or $T_1 = m(g - a) = m\ (g - g/3)$

$T_1 = 2\ mg/3 = 2 \times 2 \times 9.8/3$

$T_1 = 39.2/3 = 13$ N

45. (C) The g decreases both when we go below or above the surface of the earth. Hence, g value is maximum on the surface of the earth.

46. (A) $L = mr^2\omega$. For given m and ω, $L \alpha\ r^2$. If r is halved, the angular momentum L becomes one-fourth.

47. (A) Heat supplied $dQ = nC_pdT$

Heat used for work = dW = nRdT

$\frac{dW}{dQ} = \frac{R}{C_P} = \frac{C_P - C_v}{C_P} = 1 - \frac{C_v}{C_P}$

$= \left(1 - \frac{1}{\gamma}\right) = \left(1 - \frac{3}{4}\right) = \frac{1}{4} \times 100 = 25\%$

48. (B) Displacement equation of S.H.M is $y = a \sin \omega t$.

Therefore velocity is obtained by differentiating it.

$(v) = \frac{dv}{dt} = a\omega \cos \omega t = a\omega\sqrt{1 - \sin^2 \omega t}$

$= a\omega\sqrt{1 - \frac{y^2}{a^2}} = \omega\sqrt{a^2 - y^2}$

49. (D) Let a be the radius of sphere.

Mass of sphere $= \frac{4\pi}{3}a^3d$

Weight = $= \frac{4\pi}{3}a^3dg$

Upward thrust due to liquid w $= \frac{4}{3}\pi a^3\rho g$

Resultant weight = $\frac{4}{3}\pi a^3 g\ (d - \rho)$

$\therefore$ Resultant acceleration = $\frac{\text{force}}{\text{mass}}$

$= \frac{\frac{4}{3}\pi a^3 g\ (d - \rho)}{\frac{4}{3}\pi a^3 d}$

$= \frac{g(d - \rho)}{d}$

50. (A) Energy stored per unit volume

$= \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times \text{stress} \times \frac{\text{stress}}{Y} = \frac{1}{2}\frac{S^2}{Y}$

51. (A) $P = \frac{2S}{r} = \frac{4S}{D} = \frac{4 \times 0.072}{1.2 \times 10^{-3}} = 240\,N\,m^{-2}$

52. (B) Using the relation, $S = ut = \frac{1}{2}at^2$,

we have, $h = u\cos\theta t_1 - \frac{1}{2}gt_1^2$

and $h = u\cos\theta t_2 - \frac{1}{2}gt_2^2$

or $u\cos\theta \times 1 - \frac{1}{2} \times 9.8 \times 1^2$

$= u\cos\theta \times 3 - \frac{1}{2} \times 9.8 \times 3^2$

or $u\cos\theta(3 - 1) = 4.9 \times 8$

$$u\cos\theta = \frac{4.9\times 8}{2} = 4.9\times 4 = 19.6\,\text{m/s}$$

Maximum height

$$= \frac{u^2\cos^2\theta}{2g} = \frac{(19.6)^2}{2\times 9.8} = 19.6\text{ m}$$

53. (D) $\omega t + \phi_0 = \frac{2\pi}{T}\times t\left(\text{since } \omega = \frac{2\pi}{T}\right)$

both t, T represent time.

Hence, $\frac{2\pi t}{T}$ is dimensionless

ϕ_0 – angle so, dimensionless

$\therefore$ $\omega t + \phi_0$ has no dimensions but has certain magnitude or dimensionally it can be represented as $M^0L^0T^0$.

54. (C) Lengths of the two inclined planes are

$$l_1 = \frac{h}{\sin\theta_1} \text{ and } l_2 = \frac{h}{\sin\theta_2}$$

Acceleration of the block down the two planes are:

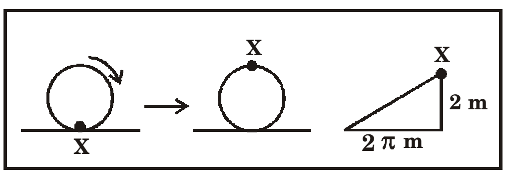
$a_1 = g\sin\theta_1$ and $a_2 = g\sin\theta_2$

$$\text{As } l_1 = \frac{1}{2}a_1t_1^2 \text{ and } l_2 = \frac{1}{2}a_2t_2^2$$

$$\therefore \frac{l_1}{l_2} = \frac{a_1t_1^2}{a_2t_2^2} \text{ or } \frac{t_2^2}{t_1^2} = \frac{a_1l_2}{a_2l_1} = \frac{g\sin\theta_1}{g\sin\theta_2}\times\frac{\sin\theta_1}{\sin\theta_2}$$

$$\therefore \frac{t_2}{t_1} = \frac{\sin\theta_1}{\sin\theta_2}$$

55. (B)



In half rotation, point 'X' has moved horizontally $\frac{\pi d}{2} = \pi r = \pi\times 2 = 2\pi$ m (since r = 2 m)

In the same time, it has moved vertically a distance which is equal to its diameter = 2r = 4 m

Therefore, displacement of

$$X = \sqrt{(2\pi)^2 + 4^2} = 2\sqrt{\pi^2 + 4}\text{ m}$$

56. (B) Work done = mg $\times$ h = weight $\times$ height

57. (A) $T = \frac{2\pi R}{v_0} = \frac{2\pi R}{\sqrt{gR}} = 2\pi\sqrt{\frac{R}{g}}$

It is independent of the mass of satellite.

58. (C) As velocities are exchanged on perfectly elastic collision, therefore, masses of two objects must be equal.

Therefore $\frac{m_a}{m_b} = 1$ (or) $m_a = m_b$

59. (B) Applying theorem of parallel axes,

$$I = I_0 + M(L/4)^2 = \frac{ML^2}{12} + \frac{ML^2}{16} = \frac{7}{48}ML^2$$

60. (C) Because water level is higher on the surface exposed to atmosphere, P_1 (pressure of confined air) is more than P_2 (atmospheric pressure).

61. (C) The specific heat at constant pressure is more than that at constant volume because additional work is needed to be done for allowing expansion of gas at constant pressure.

62. (B) Terminal velocity $v \propto r^2$. since volume $V = (4/3)\pi r^3$. Therefore, when the volume becomes 8 times, the radius is doubled. Hence velocity becomes 4 times = 4 $\times$ 1 m s^{-1} = 4 m s^{-1}

63. (A) Let massof ice be x

Latent heat of ice = 80 cal g^{-1}

Heat required by ice to convert to water

= x $\times$ 80

Heat lost by water = 160 $\times$ 20

According to principle of calorimetry,

Heat gained =Heat lost

x $\times$ 80 = 160 $\times$ 20

$$x = \frac{160\times 20}{80} = 40\,g$$

64. (A) For adiabatic compression, PV^γ = constant,

Hence, $dp = -\gamma p\frac{dV}{V}$ ...(1)

For isothermal compression, PV = constant, hence dp = $p\frac{dV}{V}$ ....(2)

Ratio (1) to (2) = γ

65. (B) Let the frequency of first tuning fork be f.

The frequency of other tuning forks are

(f – 3), (f – 2 $\times$ 3), ...(f – 17 $\times$ 3), ...(f – 25 $\times$ 3)

As per given condition

f = 2 (f – 25 $\times$ 3) or f = 25 $\times$ 6 = 150 Hz

The frequency of 18th fork

= f –17 $\times$ 3 = 150 – 51 = 99 Hz.

Chemistry

66. (C) The melting and boiling points of alkali metals decrease down the group because increase in the atomic size in going from the top to the bottom in the group. Hence, the melting point of Cs is lower than that of Na and their salts too due to their difference in lattice arrangements.

67. (A) Molecular speeds :

U_{rms} (root mean square speed) = $\sqrt{\frac{3RT}{M}}$

U_{av} (average speed) = $\sqrt{\frac{8RT}{\pi M}}$

U_{mp} (most probable speed) = $\sqrt{\frac{2RT}{M}}$

Thus, $U_{rms} : U_{av} : U_{mp} = 1 : 0.92 : 0.82$

$\Rightarrow U_{rms} > U_{av} > U_{mp}$

68. (B) Let 2nd oxide be M_xO_y, At wt of M be M.

1st oxide | **2nd oxide**

As in MO ; 50% is M

At wt of M = At wt of O

$\Rightarrow M = 16$

$\cancel{2}40 = \frac{\cancel{16}x}{\cancel{16}x + \cancel{16}y} \times \cancel{100}^{5}$

$2x + 2y = 5x$

$2y = 3x$

$\frac{x}{y} = 2 : 3$

$\therefore M_xO_y = M_2O_3$

69. (D) Temperature of the reaction, conditions like constant volume or constant pressure and the method by which change is brought about are the factors that influences the enthalpy of a reaction.

70. (D) On electrolysis of ionic hydride which contain H^- (hydride) ion, a very strong Bronsted base, it reacts with water, liberating H_2 gas at anode which escapes out. H^- is an anion and moves towards anode.

Eg: $NaH + H_2O \rightarrow NaOH + H_2\uparrow$

71. (D) ΔE for Li^{+2}ion = $E_2 - E_1$ is maximum as $E_nLi^{+2} = E_nH \times Z^2$

72. (D) The oxidation number of the various elements involved in this reaction are as follows:

$Cl_2 + 2Br^- \rightarrow 2Cl^- + Br_2$

Here, $Cl_2^{(0)} \rightarrow \overset{(-1)}{Cl}$. So, Cl_2 is the oxidant (oxidising agent) and $\overset{(-1)}{Br^-} \rightarrow Br_2^{(0)}$. So, Br^{-1} is the reductant.

73. (C) Brine solution is taken in a saturation tower of the plant. Ammonia gas containing little amount of CO_2 is passed into it, first ammonium reacts with CO_2 to form ammonium carbonate. This ammonium carbonate renders precipitation of impurities of magnesium, calcium and iron as carbonates.

$2NH_3 + H_2O + CO_2 \rightarrow (NH_4)_2CO_3$

74. (D)

$$\overset{1}{C}H_3 - \underset{}{\overset{OH}{\overset{|}{\overset{2}{C}}}}H - \overset{3}{C}H_2 - \overset{C_2H_5}{\overset{|}{\overset{4}{C}}}H = \overset{5}{C}H - \overset{6}{C}H_2$$

Hence, IUPAC name of the compound is 4–ethyl hex–4–en–2–Ol

75. (B) Aqueous solution of borax is basic as it turns red litmus paper blue.

76. (C)

$CH_3 - CH_2 - Br + 2Na + Br - CH_2 - CH_3$

$\rightarrow \underset{n-Butane}{CH_3 - CH_2 - CH_2 - CH_3} + 2NaBr$

77. (B) CCl_4 has regular tetrahedral structure as the molecules have four bond pairs around its central atom. In CCl_4, the four bond dipoles are oriented tetrahedrally to neutralise each other. As a result, the net dipole moment of the molecule is zero.

78. (C) In pure water, $[H^+] = [OH^-] = 10^{-7}$ mol lit^{-1} On addition of NH_3 (a base) to pure water, the concentration of hydronium ion decreases because in pure water, $2H_2O \rightarrow H_3O^+ + OH^-$, on addition of NH_3 it accepts hydronium ions from the solution or donates hydroxide ions to the solution. Thus, raises pH.

79. (B) $\frac{r_1}{r_2} = \frac{x/5}{x/t} = \sqrt{\frac{M_{gas}}{M_{He}}} = \sqrt{\frac{M_{gas}}{4}}$

$\frac{t}{5} = \sqrt{\frac{M_{gas}}{4}}$

If t = 20 s

$\frac{20}{5} = \sqrt{\frac{M_{gas}}{4}} \Rightarrow 16 \times 4 = M_{gas}$

$M_{gas} = 64$, Therefore SO_2 gas.

80. (D) For an adiabatic expansion of an ideal gas undergoing reversible process (i.e. no entropy generation) is:

PV^{γ} = constant or $P^{1-\gamma}T^{\gamma}$ = constant

(or) $TV^{\gamma-1}$ = constant

81. (C) Gypsum is sparingly soluble in water on heating it carefully at 100 ^{0}C, it partially loses water of crystallisation and becomes **$CaSO_4.\frac{1}{2}H_2O$**.

82. (C) Silicate is a metal derivative of silicic acid which contains silicate as anion. Silicon is a synthetic organo – silicon polymer, R_2SiO as repeat unit.

83. (A)

$$MnO_4 \rightarrow Mn^{+2} \qquad C_2O_4^{-2} \rightarrow 2CO_2$$

$$MnO_4^- \rightarrow 4Mn^{+2} + 4H_2O$$

$$MnO_4^- + 8H^+ \rightarrow Mn^{+2} + 4H_2O$$

$$MnO_4^- + 8H^+ \rightarrow 5e^- \rightarrow Mn^+ + 4H_2O$$

$$2MnO_4^- + 16H^+ \rightarrow 10e^- \rightarrow Mn^+ \quad +8H_2O + 10CO_2 + 5C_2O_4^{-2}$$

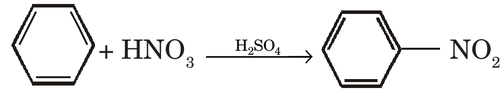
84. (D) Hydrogen bond is formed between the molecules in which hydrogen atoms are linked to an atom of highly electro negative element and atom of electronegative element should be small. In liquid HCl, chlorine is not highly electronegative to form H-bond.

85. (D) In 98 g of H_2SO_4 three are 1 g atoms of S. In 4.9 g H_2SO_4 has g atom of S

$$= \frac{4.9}{98} = 0.05 \text{ g atoms}$$

86. (A) Lithium carbonate is the least soluble alkali metal. As lithium carbonate is more covalent in nature than other carbonates of alkali metals down the group.

87. (A) In electrophilic (aromatic) nitration, aromatic organic compounds are nitrated via an electrophilic aromatic substitution mechanism involving the attack of the electron rich benzene ring by the nitronium ion.



88. (D) The orbitals described by the set of quantum numbers in given options is:

4s, 3p ; 3d, 3s

Energy of these orbitals follows the order

3d > 4s > 3p > 3s

$\therefore$ Subshell 3s has the least energy.

89. (A)

	$2HI$	$\rightleftharpoons$	H_2	I_2
initial conc.	1		0	0
Eq.conc.	$1-\frac{1}{3}=\frac{2}{3}$		$\frac{1}{6}$	$\frac{1}{6}$

K_c for $2HI \rightleftharpoons H_2 + I_2$

$$K_c = \frac{[H_2][I_2]}{[HI]^2}$$

$$\Rightarrow K_c = \frac{\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)}{\left(\frac{2}{3}\right)^2} = \frac{1}{36} \times \frac{9}{4} = \frac{1}{16}$$

90. (B) Photo chemical fog is an yellowish colour of NO_2 formed by burning of oil, coal, gas in vehicles and industries. The main cause for arise of this smog is due to oxides of nitrogen (NO and NO_2).